NAG C Library Function Document

nag_dsytri (f07mjc)

1 Purpose

nag_dsytri (f07mjc) computes the inverse of a real symmetric indefinite matrix A, where A has been factorized by nag_dsytrf (f07mdc).

2 Specification

```
void nag_dsytri (Nag_OrderType order, Nag_UploType uplo, Integer n, double a[],
     Integer pda, const Integer ipiv[], NagError *fail)
```

3 Description

To compute the inverse of a real symmetric indefinite matrix A, this function must be preceded by a call to nag_dsytrf (f07mdc), which computes the Bunch–Kaufman factorization of A.

If **uplo** = **Nag_Upper**, $A = PUDU^TP^T$ and A^{-1} is computed by solving $U^TP^TXPU = D^{-1}$ for X.

If **uplo** = **Nag_Lower**, $A = PLDL^TP^T$ and A^{-1} is computed by solving $L^TP^TXPL = D^{-1}$ for X.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates how A has been factorized as follows:

if **uplo** = **Nag_Upper**, $A = PUDU^TP^T$, where U is upper triangular;

if **uplo** = **Nag_Lower**, $A = PLDL^TP^T$, where L is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

4: **a**$[dim]$ – double *Input/Output*

Note: the dimension, dim, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.

On entry: details of the factorization of A, as returned by nag_dsytrf (f07mdc).

On exit: the factorization is overwritten by the n by n symmetric matrix A^{-1}. If **uplo** $=$ **Nag_Upper**, the upper triangle of A^{-1} is stored in the upper triangular part of the array; if **uplo** $=$ **Nag_Lower**, the lower triangle of A^{-1} is stored in the lower triangular part of the array.

5: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **a**.

Constraint: **pda** $\geq \max(1, \mathbf{n})$.

6: **ipiv**$[dim]$ – const Integer *Input*

Note: the dimension, dim, of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

On entry: details of the interchanges and the block structure of D, as returned by nag_dsytrf (f07mdc).

7: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **pda** = ⟨*value*⟩.
Constraint: **pda** > 0.

NE_INT_2

On entry, **pda** = ⟨*value*⟩, **n** = ⟨*value*⟩.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

NE_SINGULAR

The block diagonal matrix D is exactly singular.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed inverse X satisfies a bound of the form

if **uplo** $=$ **Nag_Upper**, $|DU^TP^TXPU - I| \leq c(n)\epsilon(|D||U^T|P^T|X|P|U| + |D||D^{-1}|)$;

if **uplo** $=$ **Nag_Lower**, $|DL^TP^TXPL - I| \leq c(n)\epsilon(|D||L^T|P^T|X|P|L| + |D||D^{-1}|)$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogues of this function are nag_zhetri (f07mwc) for Hermitian matrices and nag_zsytri (f07nwc) for symmetric matrices.

9 Example

To compute the inverse of the matrix A, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

Here A is symmetric indefinite and must first be factorized by nag_dsytrf (f07mdc).

9.1 Program Text

```
/* nag_dsytri (f07mjc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer i, j, n, pda;
  Integer exit_status=0;
  Nag_UploType uplo_enum;
  Nag_MatrixType matrix;

  NagError fail;
  Nag_OrderType order;
  /* Arrays */
  char    uplo[2];
  Integer *ipiv=0;
  double  *a=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
  order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07mjc Example Program Results\n\n");
  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%*[^\n] ", &n);
#ifdef NAG_COLUMN_MAJOR
  pda = n;
#else
  pda = n;
#endif

  /* Allocate memory */
```

```
  if ( !(ipiv = NAG_ALLOC(n, Integer)) ||
       !(a = NAG_ALLOC(n * n, double)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A from data file */
  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    {
      uplo_enum = Nag_Lower;
      matrix = Nag_LowerMatrix;
    }
  else if (*(unsigned char *)uplo == 'U')
    {
      uplo_enum = Nag_Upper;
      matrix = Nag_UpperMatrix;
    }
  else
    {
      Vprintf("Unrecognised character for Nag_UploType type\n");
      exit_status = -1;
      goto END;
    }

  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= n; ++j)
            Vscanf("%lf", &A(i,j));
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = 1; j <= i; ++j)
            Vscanf("%lf", &A(i,j));
        }
      Vscanf("%*[^\n] ");
    }

  /* Factorize A */
  f07mdc(order, uplo_enum, n, a, pda, ipiv, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07mdc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Compute inverse of A */
  f07mjc(order, uplo_enum, n, a, pda, ipiv, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07mjc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print inverse */
  x04cac(order, matrix, Nag_NonUnitDiag, n, n, a, pda,
         "Inverse", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04cac.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
```

```
 END:
  if (ipiv) NAG_FREE(ipiv);
  if (a) NAG_FREE(a);
  return exit_status;
}
```

9.2 Program Data

```
f07mjc Example Program Data
  4                           :Value of N
  'L'                         :Value of UPLO
  2.07
  3.87  -0.21
  4.20   1.87   1.15
 -1.15   0.63   2.06  -1.81   :End of matrix A
```

9.3 Program Results

```
f07mjc Example Program Results

 Inverse
             1           2           3           4
 1      0.7485
 2      0.5221     -0.1605
 3     -1.0058     -0.3131      1.3501
 4     -1.4386     -0.7440      2.0667      2.4547
```